trated between center conductor and lower ground plane, i.e., in the dielectric board, and the velocity goes down.

Finally, the thickness of the dielectric board was changed, again for the three preceding lines. The positions of the center conductor and of the upper edge of the board were kept constant, and the thickness was varied by changing the location of the lower edge of the board. The results for d values of $d=18$, 24, and 30 mils are shown in Fig. 6. The impedance and phase velocity decrease as expected with increasing thickness.

The impedance slope of these curves indicates a Z versus d dependence of

0.39 Ω/mil for the line width $w=72$ mils,
0.32 Ω/mil for the line width $w=120$ mils,
and
0.19 Ω/mil for the line width $w=192$ mils.

As the impedance ratios of the three lines are 1:0.76:0.56 and the Z versus d dependences are 1:0.82:0.49, it follows that a change in d is about equally critical for lower and higher ohmic lines within the range of consideration.

The fractional velocity variation is the same as the fractional impedance variation.

The accuracy of these calculations for Z is typically around 1 percent but not worse than 2 percent, for v/v_0 typically around 0.5 percent but not worse than 1 percent.

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Realizations of a Duo-Pole Branch of an Elliptic-Function Bandstop Filter

This correspondence illustrates six realizations of the TEM line (transformed) equivalent of the LC network A of Fig. 1. Network A is here taken to represent a shunt branch of a low-pass elliptic-function ladder filter [1]. Richards' transformation [2] converts a lumped element low-pass filter to a transmission-line bandstop filter [3]–[6]. Each filter element, L or C, is then replaced by a short- or open-circuited quarter-wave stub. Thus, network A is transformed to network B, with parameters as defined in Fig. 1. The six stripline and reentrant slabline networks C-H are equivalent to network B and are well suited for microwave filters. The characteristic impedances of the lines in networks C, D, E, and H are given in Fig. 1, and the coupled-line impedances of networks F and G are given in Schiffman and Matthaei [5] and Schiffman [7]. Although networks F and G are shown as cascaded sections [5], [7] (not duo-pole type), here they are shunt-connected with the far terminals open circuited. In networks B-D, line Z_1 is short circuited and line Z_2 is open circuited at its far end, and lines Z_1 and Z_2 are in series with each other at their near ends. In networks E and H, Z_2' is open circuited and in cascade with Z_1'.

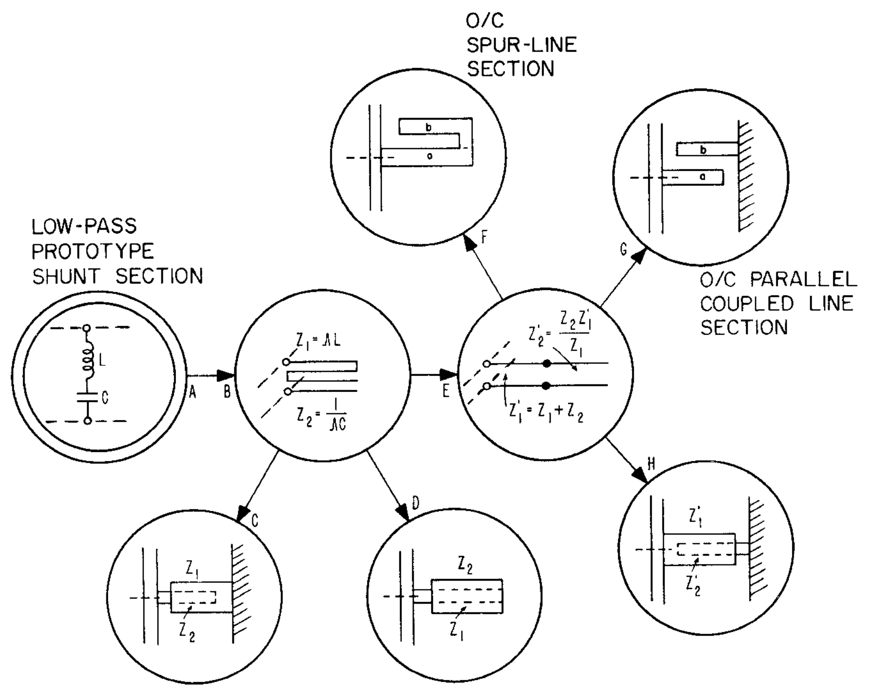

Fig. 1. Stripline and reentrant slabline realizations of a shunt duo-pole branch of an elliptic-function bandstop filter. Here, $\Lambda = \omega_1' \tan[(\pi/2)(\omega_0 - \omega_1)/\omega_0)]$ where ω_1 and ω_1' are corresponding frequencies (usually taken as band-edge frequencies) in the bandstop and low-pass frequency domains, and ω_0 is center of stopband.

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Characteristic Impedance of Multifin Transmission Lines

Several years ago, the author had occasion to investigate the characteristic impedance properties of a TEM transmission line of unusual cross section. Specifically, the cross section consisted of a round outer conductor with a center conductor composed of a number of thin fins symmetrically positioned about the axis of the line. Examples of this general class of cross section are illustrated in Fig. 1 for the cases of two, three, four, and six fins. Solutions for the characteristic impedances of this type of configuration were obtained by an interesting series of conformal transformations that mapped the multifin line geometry into that of a symmetric strip transmission line. Since the characteristic impedance of the latter is well known, curves can readily be generated for the multifin line impedance.

The basic steps in the mapping process are outlined in Fig. 2. First, geometries having other than two fins are mapped into the two-fin case by applying the transformation

$$z' = z^{n/2} \tag{1}$$

where n is the number of fins in the given geometry (z plane). Since this transformation maps $2/n$ of the multifin line space into the entire space of the two-fin line, the effect will be to establish the relations

$$Z_n = \frac{2}{n} Z_2 \tag{2}$$

when

$$\frac{r_n}{R_n} = \left(\frac{r_2}{R_2}\right)^{2/n} \tag{3}$$

where Z_n, r_n, R_n and Z_2, r_2, R_2 are the characteristic impedance, fin radial dimension, and shield radius of the n-fin and two-fin lines, respectively. Note that the z and z' planes are normalized so that the shield lies on the unit circle.

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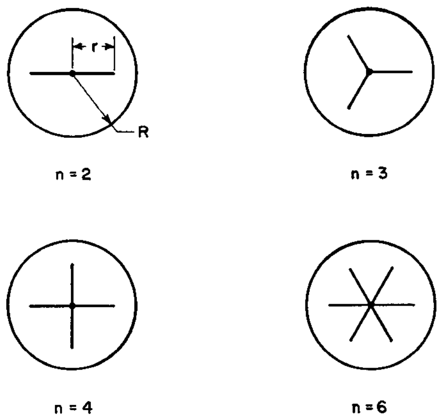

Fig. 1. Cross sections of n-fin transmission lines.

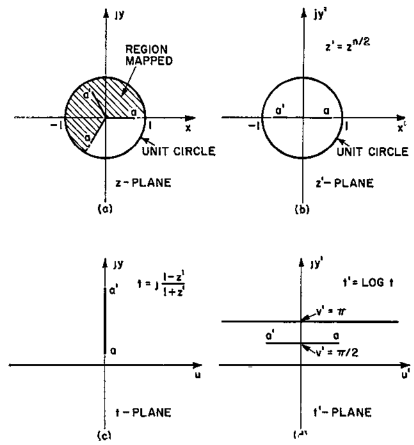

Fig. 2. Conformal mapping transformations. (a) Initial n-fin line ($n=3$ illustrated). (b) Two-fin line. (c) Intermediate transformation. (d) Strip transmission line.

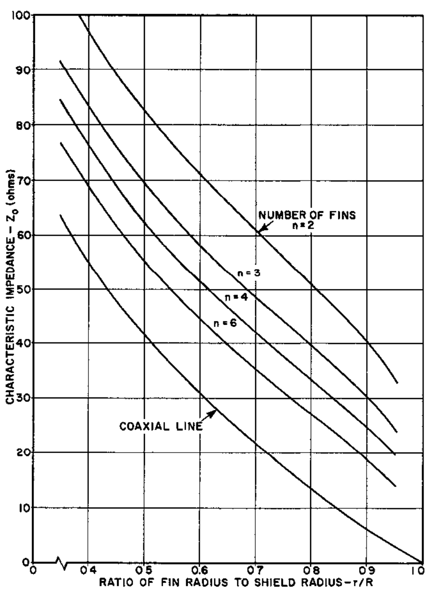

Fig. 3. Characteristic impedance of multifin lines.

The next step is to apply the transformation

$$t = j\frac{1-z'}{1+z'} \tag{4}$$

to the two-fin configuration. This transformation maps the unit circle of the z' plane onto the real axis of the t plane, and maps the fin on the real axis of the z' plane onto a portion of the imaginary axis of the t plane, as shown in Fig. 2(c). Finally, the t plane geometry is converted into a symmetric strip transmission line by the transformation

$$t' = \log t. \tag{5}$$

This relationship maps the positive real axis of the t plane onto the real axis of the t' plane, the negative real axis of the t plane onto the real axis of the t' plane, the negative real axis of the t plane onto the line $t' = u' + j\pi$, and the positive imaginary axis of the t plane onto the line $t' = u' + j\pi/2$. The portion of the t plane imaginary axis identified with the transformed fin turns out to be symmetrically positioned in the fashion of a strip transmission line. Consequently, the characteristic impedances of the strip transmission line and two-fin line will be equal when

$$\frac{w}{b} = \frac{2}{\pi}\log\left\{\frac{1+\frac{r_2}{R_2}}{1-\frac{r_2}{R_2}}\right\} \tag{6}$$

where w is the width of the strip conductor and b the ground-plane spacing.

Equations (2), (3), and (6) have been used along with data[1] on the characteristic impedance of zero-thickness strip transmission line to generate the curve of Fig. 3 for zero-thickness multifin lines. These curves show the dependence of characteristic impedance on the ratio r/R for lines of two, three, four, and six fins. Since it might be expected intuitively that a line with a large number of fins would begin to approach the r/R values of an ordinary coaxial line, this latter case has been plotted for comparison.

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Comment on "Cylindrical Waveguides Containing Inhomogeneous Dielectric"

In the above correspondence, AhSam and Klinger[1] discussed the difficulty of obtaining complete analytic solutions for propagation of electromagnetic waves in inhomogeneous media with circular cylindrical symmetry. In order to aid the search for exact solutions, they studied the special case in which the relative permittivity, independent of the azimuthal and axial coordinates, varies inversely as the square of the radius so that two of the six field equations uncouple.

I should like to point out that AhSam and Klinger overlooked the existence of certain classes of modes. That modes for nonzero azimuthal variation (i.e., $n \neq 0$ in their notation) *must* exist, independently of the relationship among a, b, and l (the radii of the metal cylinders containing the inhomogeneous medium and the constant entering the relative permittivity), can be deduced from the following arguments. First, as the frequency is raised, the medium appears more and more uniform locally, and a high-frequency wave launched in some direction not in the r-z plane should have no difficulty propagating down the waveguide. Secondly, since the modes in any closed cylindrical waveguide form a complete set, one should be able to expand a function of θ in terms of it, so that the set must contain modes with $n \neq 0$.

Except in very special cases, azimuthally dependent modes in circular cylindrical structures are hybrid modes, since it is generally not possible to satisfy all the boundary conditions with just TE or TM. Thus, AhSam and Klinger are correct in stating that TM solutions (only f_2, g_1, and g_3 nonzero) and TE solutions (only g_2, f_1, and f_3) do not exist for $n \neq 0$. It is also true that all six components cannot exist simultaneously (except for the special relations between a, b, and l) since the two characteristic equations (13) and (18)[1] would not have simultaneous solutions. However, it is possible to have modes with nonzero f_1, f_2, f_3, g_1, and g_3, or f_1, f_3, g_1, g_2, and g_3 which satisfy all the given conditions. In the first case, f_2 is given as in (12),[1] with

$$f_1 = \frac{\beta}{\alpha k_z}\frac{d}{dr}(rf_2)$$

$$f_3 = -\frac{j}{\alpha}r\beta f_2$$

$$g_1 = \frac{-k_z}{(\omega\mu)^2}\frac{k_0{}^2 l}{\alpha}f_2$$

$$g_3 = j\frac{k_0{}^2 l}{\alpha(\omega\mu)^2}\frac{1}{r}\frac{d}{dr}(rf_2)$$

and k_z determined by (13). Similarly, a possible solution using (18) to determine k_z has g_2 as given by (16)[1] and

$$f_1 = \frac{r^2 k_z}{\alpha}g_2$$

$${}_3 = -\frac{j}{\alpha}\left[r^2\frac{dg_2}{dr} + rg_2\right]$$

$$g_1 = -\frac{\beta}{\alpha k_z}\frac{d}{dr}(rg_2)$$

$$g_3 = jr\frac{\beta}{\alpha}g_2.$$

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